

Transient properties of modified reservoir-induced transparency

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We investigate the transient response of a Λ -type system with one transition decaying to a modified radiation reservoir with an inverse square-root singular density of modes at threshold, under conditions of transparency. We calculate the time evolution of the linear susceptibility for the probe laser field and show that, depending on the strength of the coupling to the modified vacuum and the background decay, the probe transmission can exhibit behaviour ranging from underdamped to overdamped oscillations. Transient gain without population inversion is also possible depending on the system's parameters.

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It has been now well documented that quantum coherence and interference effects can modify the absorption and dispersion properties of an atomic system [1,2]. In the most common situation, that of a Λ -type three-level system, the medium becomes transparent to a probe laser field near an otherwise absorbing resonant transition. This is achieved via the application of a second laser field coupling to the linked transition. In addition to steady state studies, considerable work has been done on the transient properties of coherent phenomena such as, for example, electromagnetically induced transparency [3,4], gain (or lasing) without inversion [5–7] and coherent population trapping [8,9].

As has been recently shown [10], transparency can occur in the steady state absorption of a Λ -type system when one of the atomic transitions is coupled to a modified radiation reservoir having a threshold with an inverse square-root dependence of the density of modes, $\rho(\omega) = \Theta(\omega - \omega_g) / (\pi\sqrt{\omega - \omega_g})$, with Θ being the Heaviside step function and ω_g being the gap frequency. Such a density of modes can be found near thresholds in waveguides [11,12], in microcavities [13,14], and near the edge of a photonic band gap material which is described by an isotropic model [15–19]. We also note that there is current interest in coherent phenomena which occur in modified reservoirs having relatively weak modal densities where the Born and Markov approximations can be applied [20,21].

It is known that coherence effects can take a considerable time to be set up [22], and the purpose of the present work is to investigate this question when structured radiation reservoirs are employed. In this article we study the transient behaviour of the absorption of a Λ -type system, similar to the one used in ref. [10], with one of the atomic transitions spontaneously coupled to a frequency-dependent reservoir which displays the above mentioned inverse square-root behaviour in its density of modes. We show that the rate at which the atomic medium becomes transparent to the probe field depends crucially on both the background decay rate of the upper atomic level and the strength of the coupling to the modified vacuum modes. We also find that, under certain conditions, the system can exhibit transient gain without

inversion.

The atomic system under consideration is shown in figure 1. It consists of three atomic levels in a Λ -type configuration. The atom is assumed to be initially in state $|0\rangle$. The transition $|1\rangle \leftrightarrow |2\rangle$ is taken to be near-resonant with a frequency-dependent reservoir, while the transition $|0\rangle \leftrightarrow |1\rangle$ is assumed to be far away from the gap and is treated as a free space transition. The dynamics of the system can be described using a probability amplitude approach. The Hamiltonian of the system, in the interaction picture and the rotating wave approximation, is given by (we use units such that $\hbar = 1$),

$$H = \left[\Omega e^{i\delta t} |0\rangle\langle 1| + \sum_{\mathbf{k},\lambda} g_{\mathbf{k},\lambda} e^{-i(\omega_{\mathbf{k}} - \omega_{12})t} |1\rangle\langle 2| a_{\mathbf{k},\lambda} + \text{H.c.} \right] - i\frac{\gamma}{2} |1\rangle\langle 1|. \quad (1)$$

Here, $\Omega = -\boldsymbol{\mu}_{01} \cdot \boldsymbol{\epsilon} E$ is the Rabi frequency, with $\boldsymbol{\mu}_{nm}$ being the dipole matrix element of the $|n\rangle \leftrightarrow |m\rangle$ transition. The unit polarization vector and the electric field amplitude of the probe laser field are denoted by $\boldsymbol{\epsilon}$ and E respectively. Also, $\delta = \omega - \omega_{10}$ is the laser detuning from resonance with the $|0\rangle \leftrightarrow |1\rangle$ transition, where $\omega_{nm} = \omega_n - \omega_m$ and ω_n is the energy of state $|n\rangle$ and ω is the probe laser field angular frequency. In addition, γ denotes the background decay which can be treated as a Markovian process and is included as a simple loss term from the excited state $|1\rangle$. Finally, $g_{\mathbf{k},\lambda} = -i\sqrt{2\pi\omega_{\mathbf{k}}/V} \boldsymbol{\epsilon}_{\mathbf{k},\lambda} \cdot \boldsymbol{\mu}_{12}$ where V is the quantization volume, $\boldsymbol{\epsilon}_{\mathbf{k},\lambda}$ is the unit polarization vector, $a_{\mathbf{k},\lambda}$ is the photon annihilation operator and $\omega_{\mathbf{k}}$ is the angular frequency of the $\{\mathbf{k}, \lambda\}$ mode of the modified radiation reservoir vacuum field.

The wavefunction of the system, at a specific time t , can be expanded in terms of the ‘bare’ eigenvectors such that

$$|\psi(t)\rangle = b_0(t)|0, \{0\}\rangle + b_1(t)e^{-i\delta t}|1, \{0\}\rangle + \sum_{\mathbf{k},\lambda} b_{\mathbf{k},\lambda}(t)|2, \{\mathbf{k}, \lambda\}\rangle, \quad (2)$$

and $b_0(t=0) = 1$, $b_1(t=0) = 0$, $b_{\mathbf{k},\lambda}(t=0) = 0$. We substitute Eqs. (1) and (2) into the time-dependent

Schrödinger equation and obtain the time evolution of the probability amplitudes as

$$i\dot{b}_0(t) = \Omega b_1(t), \quad (3)$$

$$i\dot{b}_1(t) = \Omega b_0(t) - \left(\delta + i\frac{\gamma}{2}\right) b_1(t) - i \int_0^t dt' K(t-t') b_1(t'), \quad (4)$$

$$i\dot{b}_{\mathbf{k},\lambda}(t) = g_{\mathbf{k},\lambda} e^{i(\omega_{\mathbf{k}} - \omega_{12} - \delta)t} b_1(t), \quad (5)$$

with the kernel

$$K(t-t') = \sum_{\mathbf{k},\lambda} g_{\mathbf{k},\lambda}^2 e^{-i(\omega_{\mathbf{k}} - \omega_{12} - \delta)(t-t')} \approx \beta^{3/2} \int d\omega \rho(\omega) e^{-i(\omega - \omega_{12} - \delta)(t-t')}, \quad (6)$$

and β being the atom-modified reservoir resonant coupling constant. All the coupling constants ($g_{\mathbf{k},\lambda}$, β , Ω) are assumed to be real, for simplicity.

The time evolution of the absorption and dispersion properties of the system are determined by, respectively, the imaginary and real parts of the time-dependent linear susceptibility $\chi(t)$. In our case, the susceptibility can be expressed as [3]

$$\chi(t) = -\frac{4\pi\mathcal{N}|\mu_{01}|^2}{\Omega(z,t)} b_0(t) b_1^*(t), \quad (7)$$

with \mathcal{N} being the atomic density. The solution of Eqs. (3) and (4) is obtained by means of time-dependent perturbation theory [5,10]. We assume that the laser-atom interaction is very weak ($\Omega \ll \beta, \gamma$) so that $b_0(t) \approx 1$ for all times. Then, Eqs. (3) and (4) reduce to

$$i\dot{b}_1(t) \approx \Omega - \left(\delta + i\frac{\gamma}{2}\right) b_1(t) - i \int_0^t dt' K(t-t') b_1(t'). \quad (8)$$

We further assume that $\Omega(z,t)$ is approximately constant in the medium and with the use of the Laplace transform we obtain from Eq. (8)

$$\tilde{b}_1(s) = \frac{\Omega}{s \left[\delta + i\gamma/2 + i\tilde{K}(s) + is \right]}, \quad (9)$$

where $\tilde{b}_1(s) = \int_0^\infty e^{-st} b_1(t) dt$, $\tilde{K}(s) = \int_0^\infty e^{-st} K(t) dt$. The amplitude $b_1(t)$ is given by the inverse Laplace transform

$$b_1(t) = \frac{1}{2\pi i} \int_{\epsilon-i\infty}^{\epsilon+i\infty} e^{st} \tilde{b}_1(s) ds, \quad (10)$$

where ϵ is a real number chosen so that $s = \epsilon$ lies to the right of all the singularities (poles and branch cut points) of function $\tilde{b}_1(s)$.

For the case of an inverse square-root singularity in the frequency-dependent reservoir density of modes $\tilde{K}(s) =$

$\beta^{3/2} e^{-i\pi/4} / \sqrt{s + i(\delta_g - \delta)}$ with $\delta_g = \omega_g - \omega_{12}$, the inverse Laplace transform of Eq. (9) yields

$$b_1(t) = \sum_{i=1}^5 \alpha_i (x_i + y_i) e^{x_i^2 t} - \sum_{i=1}^5 \alpha_i y_i \left[1 - \text{erf}(\sqrt{x_i^2 t}) \right] e^{x_i^2 t}, \quad (11)$$

where $y_i = \sqrt{x_i^2}$ and x_i are the roots of the equation

$$x^5 + c_3 x^3 + c_2 x^2 + c_1 x + c_0 = 0. \quad (12)$$

Here $c_3 = \gamma/2 - i(\delta_g + \delta')$, $c_2 = -iK_0$, $c_1 = -\delta'(\delta_g + i\gamma/2)$, $c_0 = -K_0\delta'$, $\delta' = \delta_g - \delta$, $K_0 = i\beta^{3/2} e^{-i\pi/4}$ and erf is the error function [23]. The roots of this equation are determined numerically. The expansion coefficients α_i are given by

$$\alpha_i = \frac{i\Omega x_i}{(x_i - x_j)(x_i - x_k)(x_i - x_l)(x_i - x_m)}, \quad (13)$$

with $i, j, k, l, m = 1, 2, 3, 4, 5$. Also, if $\text{Re}(x_i) > 0$ we have $y_i = x_i$, while if $\text{Re}(x_i) < 0$ we have $y_i = -x_i$, in order to keep the phase angle of x_i^2 between $-\pi$ and π [23]. In addition, if $\text{Re}(x_i) = 0$ then $\alpha_i = 0$. Therefore, at least two roots and at most three roots contribute on the solution (11) depending on the system parameters.

Within our perturbative approach, Eq. (7) yields $\chi(t) \sim -b_1^*(t)$, where $b_1(t)$ is given by Eq. (11). As has been shown in ref. [10], steady state transparency occurs for the case that $\delta = \delta_g$. This is the case that also interests us here. In figure 2 we plot the time evolution of the imaginary part of the linear, time-dependent susceptibility $[-\text{Im}(\chi(t))]$ for different values of the background decay γ and with $\delta = \delta_g = 0$. In the case that $\gamma \gg \beta$, the susceptibility is always positive (which denotes absorption in our convention), has a maximum and the steady state value is reached adiabatically. If $\gamma = \beta$, the system exhibits only absorption, however small oscillations are visible at the beginning. As γ decreases, then these oscillations become more pronounced, and small gain (or lasing) without the presence of population inversion between $|1\rangle$ and $|0\rangle$, shown by negative values of the time-dependent linear susceptibility, is found. If the background decay decreases further and reaches the regime that $\gamma \ll \beta$ the oscillations increase further, the gain without inversion increases, the interaction becomes more non-adiabatic and the steady state value is reached for very large times.

The behaviour displayed in the previous figure can be understood if the time evolution of the population of the excited state $|1\rangle$ is examined. As can be seen from figure 3, after an initial weak absorption the population of the state $|1\rangle$ can either decay smoothly to zero (for the case $\gamma \gg \beta$) or evolve by undergoing damped Rabi oscillations between states $|1\rangle$ and $|2\rangle$ due to reversible decay which arises via the interaction with the modified reservoir [15,16]. These oscillations increase as the

background decay decreases compared to the coupling strength to the frequency-dependent radiation reservoir. In such a way a time-dependent coherence between states $|1\rangle$ and $|2\rangle$ is created which is responsible for the phenomenon of transient gain without inversion shown in figure 2.

This behaviour of the system is related to the one predicted [3] and experimentally observed [4] in a typical three level Λ -type atomic system which exhibits electromagnetically induced transparency through the application of a coupling laser field. The difference in our case, is that the transparency and the transient gain without inversion occur due to the coupling to a radiation reservoir with an inverse square-root singularity of the density of modes at threshold and are not induced by an external laser field.

In summary, we have discussed the transient properties of the transparency in a Λ -type atom in which one transition spontaneously decays to a specific frequency-dependent radiation reservoir. The time evolution of the absorption and thus the way that the steady state is reached depends crucially on the background decay rate and the strength of the coupling to the modified reservoir. Transient gain without population inversion is found to exist if the coupling strength to the modified reservoir is larger than the background decay rate. We have only been concerned with the time evolution of the linear absorption properties of the medium. The time evolution of the dispersive properties of the system, which is another topic of interest [10], will be discussed separately. In such a study the simple relationship between the real part of the susceptibility and the group velocity cannot be applied (as it holds only for the steady state), and a different approach needs to be implemented.

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- [1] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997), Chapter 7.
- [2] J.P. Marangos, *J. Mod. Opt.* **45**, 471 (1998).
- [3] Y.-Q. Li and M. Xiao, *Opt. Lett.* **20**, 1489 (1995).
- [4] H.X. Chen, A.V. Durrant, J.P. Marangos and J.A. Vaccaro, *Phys. Rev. A* **58**, 1545 (1998).
- [5] S.E. Harris and J.J. Macklin, *Phys. Rev. A* **40**, 4135 (1989).
- [6] E.S. Fry, X.F. Li, D.E. Nikonov, G.G. Padmabandu, M.O. Scully, A.V. Smith, F.K. Tittel, C. Wang, S.R. Wilkinson and S.-Y. Zhu *Phys. Rev. Lett.* **70**, 3235 (1993).
- [7] Y. Zhu, *Phys. Rev. A* **55**, 4568 (1997).
- [8] P.L. Knight, M.A. Lauder and B.J. Dalton, *Phys. Rep.* **190**, 1 (1990).
- [9] E.A. Korsunsky, W. Maichen and L. Windholz, *Phys. Rev. A* **56**, 3908 (1997).
- [10] E. Paspalakis, N.J. Kylstra and P.L. Knight, *Phys. Rev. A* **60**, R33 (1999).
- [11] D. Kleppner, *Phys. Rev. Lett.* **47**, 233 (1981).
- [12] M. Lewenstein, J. Zakrzewski, T.W. Mossberg and J. Mostowski, *J. Phys. B* **21**, L9 (1988).
- [13] M. Lewenstein M. and T.W. Mossberg, *Phys. Rev. A* **37** 2048 (1988).
- [14] M.A. Rippin and P.L. Knight, *J. Mod. Opt.* **43**, 807 (1996).
- [15] S. John and T. Quang, *Phys. Rev. A* **50**, 1764 (1994); T. Quang, M. Woldeyohannes, S. John and G.S. Agarwal, *Phys. Rev. Lett.* **79**, 5238 (1997).
- [16] A.G. Kofman, G. Kurizki and B. Sherman, *J. Mod. Opt.* **41**, 353 (1994).
- [17] S.-Y. Zhu, H. Chen and H. Huang, *Phys. Rev. Lett.* **79**, 205 (1997).
- [18] S. Bay, P. Lambropoulos and K. Mølmer, *Phys. Rev. Lett.* **79**, 2654 (1997).
- [19] E. Paspalakis, D.G. Angelakis and P.L. Knight, LANL e-print quant-ph/9906093.
- [20] O. Kocharovskaya, P. Mandel and M.O. Scully, *Phys. Rev. Lett.* **74**, 2451 (1995).
- [21] M. Erhard and C.H. Keitel, submitted for publication (1999).
- [22] B.J. Dalton and P.L. Knight, *J. Phys. B* **15**, 3997 (1982).
- [23] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1980).

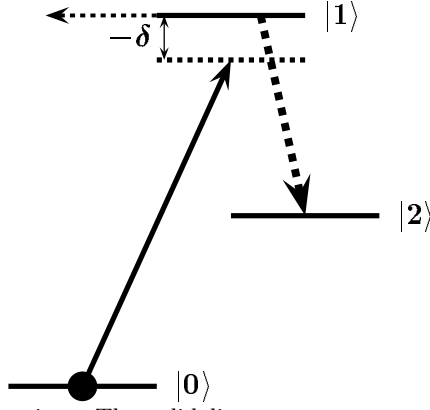


FIG. 1. The system under consideration. The solid line denotes the probe laser coupling, the thick dashed line denotes the coupling to the modified radiation reservoir and finally the thin dashed line denotes the background decay.

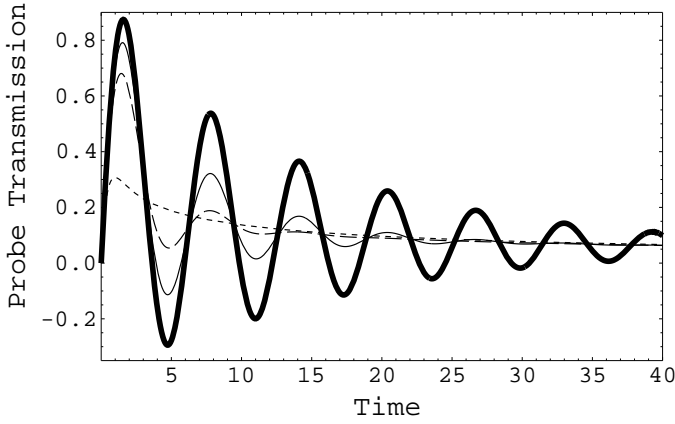


FIG. 2. The time evolution of the imaginary part of the time-dependent linear susceptibility $[-\text{Im}(\chi(t))]$ (in arbitrary units). In our notation positive (negative) values denote probe absorption (gain). The parameters used were $\delta = \delta_g = 0$ and $\gamma = 5$ (shot dashed curve), $\gamma = 1$ (long dashed curve), $\gamma = 0.5$ (thin solid curve), and $\gamma = 0.2$ (thick solid curve). All parameters are measured in units of β .

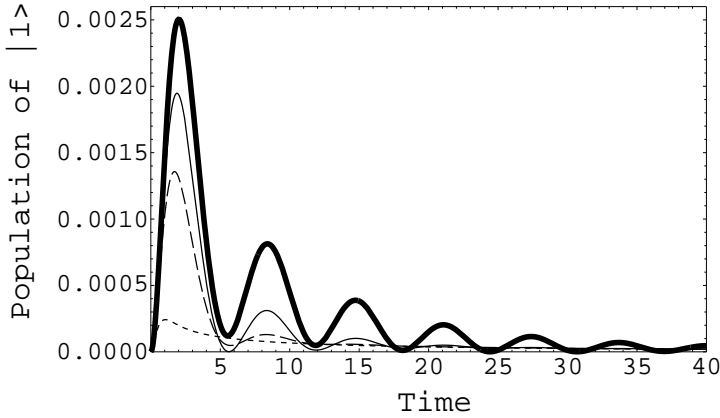


FIG. 3. The time evolution of the population of state $|1\rangle$. The parameters used are the same as in figure 2.